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# Reliability Assessment of a Bridge Structure Subjected to Chloride Attack

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## Abstract

Prediction of the service lifetime of concrete structures with respect to chloride ingress involves a number of parameters that are associated with large uncertainties. Hence, full-scale measurements are strongly in demand. This paper begins by summarizing statistical distributions based on measurements taken from the Gimsøystraumen Bridge in Norway. A large number of chloride profiles are available based on concrete coring samples, and for each of these profiles the diffusion coefficient and surface concentration (due to sea spray) are estimated. Extensive measurements of the concrete cover depth are also performed. The probability distributions are input into a prediction model for chloride concentration at the steel reinforcement. By also introducing the critical chloride concentration as a random variable, the probability of exceeding the critical threshold is determined as a function of time. To address chloride attack on the entire bridge, a system model with 90 components is introduced. Reliability updating based on observations at multiple sites along the bridge is also investigated. First-order reliability methods typically become inaccurate for large systems of this type, so an enhanced Monte Carlo simulation method is applied. It is shown that the corresponding computation time is significantly reduced compared to crude Monte Carlo methods.

**Keywords:** chloride ingress; bridge test data; system reliability; enhanced Monte Carlo method; Service lifetime

## Introduction

The first part of the present paper focuses on the estimation of the statistical properties of parameters related to the prediction of chloride ingress based on full-scale measurements. A large number of chloride profiles have been obtained based on concrete cores from the Gimsøystraumen Bridge, located in the Lofoten area in the northern part of Norway (*Fig. 1*). Profiles from 725 locations were collected from the superstructure (i.e. the north side, the underside and the south side). For the columns, sampling was performed at 168 locations.<sup>1</sup> For each of the profiles, the corresponding diffusion coefficient and the chloride surface concentration were estimated based on laboratory tests of the drilled concrete core samples from the bridge. Extensive measurements of concrete cover were also performed. The probability distributions which were estimated from the measurements are presently being employed as input into a model for the prediction of the chloride concentration at the steel reinforcement. As the input


parameters are represented in probabilistic terms, the chloride concentration accordingly becomes a stochastic quantity. The critical chloride concentration is also represented

as a random variable. As a next step, the resulting probability that the concentration at the reinforcement will exceed the critical threshold is determined as a function of time.<sup>2</sup>

In order to address the chloride attack on the entire bridge, a system model with 90 components is employed to perform reliability updating based on observations at a number of sites along the bridge. The computations are performed by application of the so-called enhanced Monte Carlo simulation method.<sup>3,4</sup>

The present study has three primary objectives: to identify the best statistical models (if any) for three of the key parameters in relation to the assessment of the chloride ingress, namely the chloride concentration at the surface, the diffusion coefficient and the concrete cover of the steel reinforcement; to investigate the covariation properties for two of these parameters, namely the surface concentration and the diffusion coefficient; and to study system effects by introducing an enhanced simulation



*Fig. 1: The Gimsøystraumen Bridge in northern Norway (Lofoten area) (Photo: Petr Šmerkl, Wikipedia )*

method while also illustrating the gain in computation time that can be achieved by the application of this method. A summary of the method used to achieve these objectives is as follows:

- Probabilistic models based on full-scale measurements of the diffusion coefficient, the surface concentration and the concrete cover are provided.
- Reliability analysis is performed at both the component and the system level.
- The effect of additional information based on monitoring and inspection is quantified in relation to the system reliability.
- Savings of computational effort by application of an enhanced Monte Carlo simulation method for evaluation of system reliability are quantified.

### Probabilistic Modelling Based on Full-Scale Measurements

The Gimsøystraumen Bridge has served as a “test bridge” for many purposes, including the assessment of different types of repair method. The objective of the present study is to assess the merits of relevant probabilistic models based on full-scale data and show how they can be applied for the purpose of lifetime assessment with respect to chloride ingress. A further aim is to illustrate how information from monitoring and inspection can serve in relation to reliability updating. In order to attain a more realistic model of the entire bridge structure, a system model is subsequently established. As computation of the corresponding system reliability as a function of time can quickly become demanding, it is also demonstrated how so-called enhanced Monte Carlo techniques can serve to make the calculation of structural reliability more efficient than the crude Monte Carlo technique.

#### Statistical Models Based on Test Data

For each of the three parameters that were measured or estimated based on the measurements (i.e. the diffusion coefficient, the surface concentration and the concrete cover), the applicability of various analytical probability distributions were tested by plotting the samples on different types of

probability paper. A ranking was performed based on the regression coefficients. As an example, a summary of the results are shown in *Table 1* for the diffusion coefficients obtained for the east side of the columns.

As observed, the Weibull distribution gives the highest regression coefficient,  $R^2$ . The measured and analytical distribution functions as plotted on Weibull probability paper are compared in *Fig. 2*. However, in general all the different distributions give quite high values for the regression coefficient—possibly with exception of the gamma distribution, which has a somewhat reduced regression coefficient of 0.79. A more direct comparison between the analytical model and the observed data is provided by considering the density function, namely the expected versus observed number of samples within each discretized interval (*Fig. 3*). The overall comparison is quite good, but with some “oscillations” around the theoretical curve.

Although the Weibull model gives the best fit for this specific case, it is found that on average the lognormal probability distribution gives the best fit. Furthermore, this model is very

convenient in relation to the Rosenblatt probability transformations which are applied in connection with the first-order reliability methods introduced later in the paper. Accordingly, the lognormal model is employed as the reference model in the following—hence, the lognormal distribution is applied for the present calculations of lifetime distributions.

The regression coefficients obtained from a similar fit of probability distributions for the *chloride surface concentration* are shown in *Table 2*. It is observed that the lognormal distribution gives the highest value for the regression coefficient. However, all the distributions have regression coefficients higher than 0.9, which in general is quite acceptable.

Measurements of the concrete cover depth were also performed, and a lognormal model was found to give the best fit. Based on the full-scale measurements and consideration of the additional parameters entering into the computation of the chloride lifetime, corresponding probabilistic models were established. The relevant parameters are defined in relation to the solution of Fick’s second law<sup>5</sup> for

Probabilistic model	Regression line	Mean	SD	Sample variance	$R^2$
Normal	$y = 1.6051x - 2.0384$	1.27	0.64	0.41	0.9815
Gamma	$y = 0.7388x - 0.6466$				0.7934
Gumbel	$y = 2.1481x - 2.1085$				0.9899
Weibull	$y = 2.3307x - 0.8316$				0.9948
Lognormal	$y = 1.9765x - 0.1641$				0.9809

Table 1: Diffusion coefficient (units in  $m^2/s \times 10^{-12}$ )

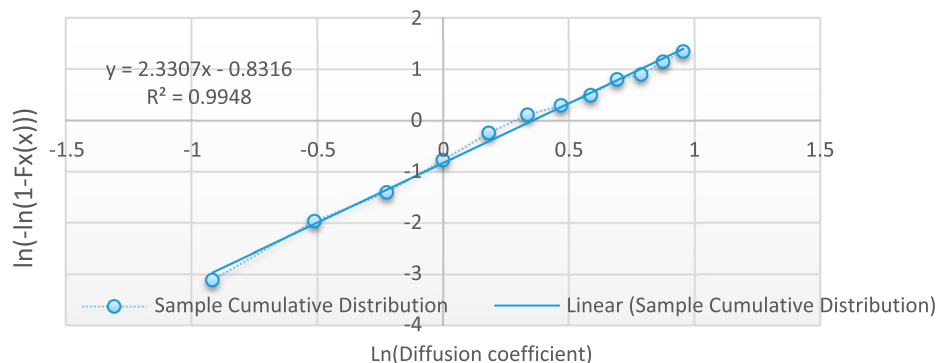


Fig. 2: Comparison between the sample distribution function and the fitted Weibull distribution for the diffusion coefficient of the east column

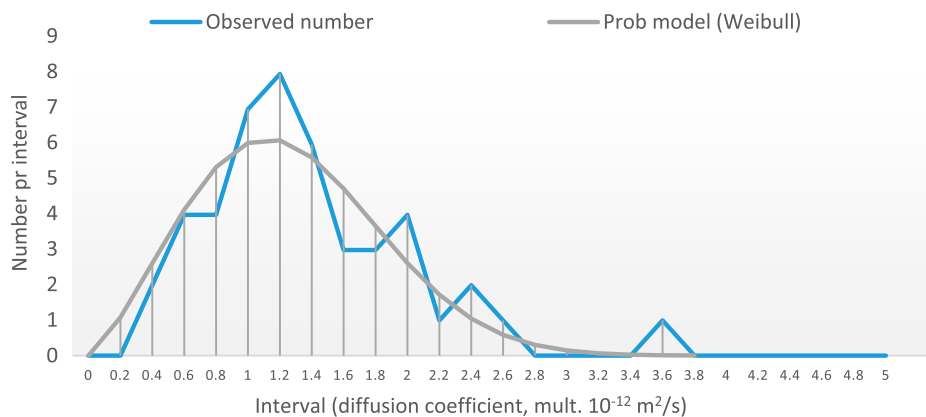


Fig. 3: Observed versus predicted number of samples for the diffusion coefficient within each interval of the east column (theoretical model based on the regression curve in Figure 2)

Probabilistic model	Regression line	Mean	SD	Sample variance	R <sup>2</sup>
Normal	$y = 3.5447x - 1.5979$	0.50	0.34	0.11	.9156
Gamma	$y = 1.4932x - 0.3422$				.9352
Gumbel	$y = 4.0841x - 1.3635$				.9716
Weibull	$y = 1.9038x + 1.2355$				.9338
Lognormal	$y = 1.4879x + 1.3571$				.9826

Table 2: Surface concentration  $C_s$  (% of concrete weight)

Statistical variable	Distribution type	Mean value	Standard deviation
Surface concentration	Lognormal	0.25% concrete weight	0.18% concrete weight
Diffusion coefficient	Lognormal	$0.88 \text{ m}^2/\text{s} \times 10^{-12}$	$0.68 \text{ m}^2/\text{s} \times 10^{-12}$
Initial concentration	Normal	0.015% concrete weight	0.0015% concrete weight
Concrete cover	Lognormal	23 mm	6 mm
Critical chloride concentration	Lognormal	0.18% concrete weight	0.06% concrete weight
Model uncertainty	Normal	1.0	0.01/0.10

Table 3: Statistical distributions for the superstructure

the chloride concentration  $c(x,t)$  at position (depth)  $x$  and time  $t$ :

$$c(x, t) = c_i + (c_s - c_i) \cdot \operatorname{erfc}\left(\frac{x}{2\sqrt{D \cdot t}}\right) \quad (1)$$

where  $c_i$  is the initial chloride concentration in the concrete,  $c_s$  is the chloride concentration at the surface, and  $D$  is the diffusion coefficient. The concentration at the position of the reinforcement is subsequently compared to the

critical chloride concentration for the onset of corrosion. The diffusion coefficient can also be represented as being time-dependent by application of the so-called alpha-factor.<sup>6,7</sup> In the present analysis the diffusion coefficient is initially assumed to be constant in time, but the effect of applying a decreasing value with time is also investigated.

The probabilistic models which are applied for the superstructure are summarized in Table 3. The model

uncertainty factor is introduced in order to account for deviations between the model predictions and the observed diffusion rates. The lowest value of the standard deviation is taken to represent lifetime calculations performed for the bridge on which the measurements were performed. The highest value could for example represent a situation where these particular data were applied for calculations of a “similar” bridge.

Corresponding models which apply to the columns are given in Table 4. As observed, both the diffusion coefficient and the surface concentration are higher for this case. However, the concrete cover also has a considerably higher value than for the superstructure.

#### Covariance Properties of the Diffusion Parameters

The covariance structure of the diffusion coefficient and the surface concentration for the north side of the bridge is also investigated. The autocorrelation function for each of these quantities is estimated based on the following expression:

$$R_{i,i+k} = \frac{1}{N-1} \cdot \sum_{i=1}^{i=N} (x_i \cdot x_{i+k}) \quad (2)$$

where  $x_i$  and  $x_{i+k}$  are the values of the relevant quantity at the cross sections  $i$  and  $i+k$  and  $N$  is the total number of cross sections, which is equal to 76 for the north side of the superstructure. The total distance between the first and last cross sections is 278.6 m, which corresponds to half the bridge length. The pairwise distance between the cross sections is more or less constant. In Eq. (2), the index  $i+k$  runs from  $k+1$  to  $k+N$ , while the measured record only is of length  $N$ . Accordingly, the values  $x_{k+N}$  are not well defined for values of  $k$  larger than zero. This is presently handled by introducing trailing zeros whenever the index exceeds  $N$ . It is also to be noted that for each cross section, three or four sample cores were extracted from the concrete. In the present calculation, the mean value of these samples is applied for each cross section.

In order to normalize the autocorrelation function, it is divided by its value at  $R_{1,1}$  (which corresponds to the estimated value of  $E(x^2)$ ). The resulting normalized function for the diffusion coefficient is shown in Fig. 4 and the

Statistical variable	Distribution type	Mean value	Standard deviation
Surface concentration	Lognormal	0.50% concrete weight	0.34% concrete weight
Diffusion coefficient	Lognormal	$1.27 \text{ m}^2/\text{s} \times 10^{-12}$	$0.64 \text{ m}^2/\text{s} \times 10^{-12}$
Initial concentration	Uniform	0.015% concrete weight	0.0015% concrete weight
Concrete cover	Lognormal	45 mm	6 mm
Critical chloride concentration	Lognormal	0.18% concrete weight	0.06% concrete weight
Model uncertainty	Normal	1.0	0.01/0.10

Table 4: Statistical distributions for the columns

function for the surface concentration is shown in Fig. 5. The horizontal axis shows the cross section increment number along the bridge, that is the value of the incremental index  $k$  in Eq. (2). It is seen that in order to make the normalized autocorrelation function for the diffusion coefficient smaller than 0.5, this increment needs to be  $k=20$  or higher. For the surface concentration, the corresponding value is  $k=15$ .

### Lifetime Probability Distribution Based on the Estimated Models

The cumulative distribution functions for chloride lifetime are obtained by

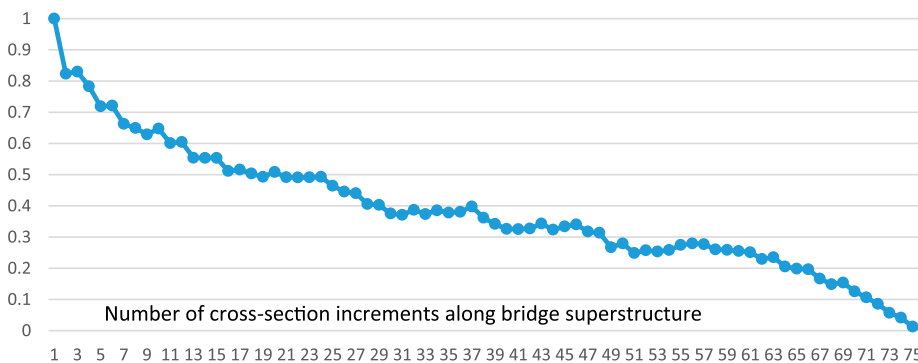


Fig. 4: Normalized autocorrelation function for the diffusion coefficient of the north side of the bridge superstructure

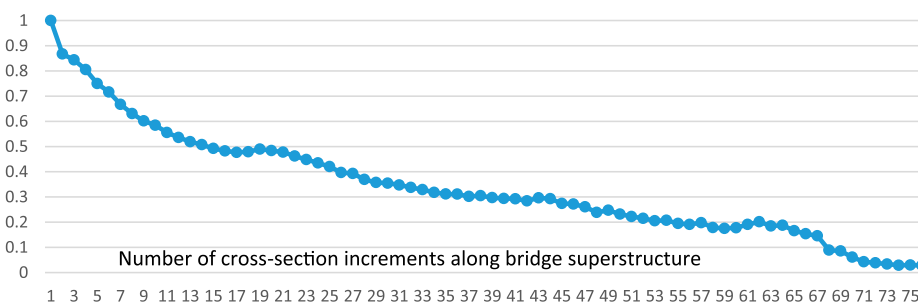


Fig. 5: Normalized autocorrelation function for the surface chloride concentration of the north side of the bridge superstructure

calculating probabilities of the type  $P$  (chloride concentration at reinforcement at time  $t <$  critical chloride concentration). These probabilities are computed repeatedly for a number of different values of the time parameter by application of the so-called first-order reliability methods.<sup>8,9</sup>

The cumulative probability distribution that results from the reliability analysis based on the input data given in Table 3 (superstructure) is shown in Fig. 6, while the corresponding probability density function is obtained by numerical differentiation and given in Fig. 7. As observed, the peak of the latter occurs for a lifetime of six years. However, the shape of the upper tail

is such that it decays very slowly. This implies a large standard deviation for the lifetime. This is also reflected by the slow rise of the cumulative distribution function, which obtains a value of 0.4 for a duration of 80 years. Accordingly, the probability of the lifetime being less than this value is 40%.

The effect of including a probabilistic time-varying diffusion is accounted for by introducing the alpha parameter as discussed above, which is presently achieved by modelling it as a random variable. The mean value is taken to be 0.4 and the standard deviation is 0.1; a lognormal distribution is assumed to apply. The peak of the density function is also now located at six years; however, the peak is now much smaller than in Fig. 6. The upper tail of the density is also higher, and the corresponding distribution function in Fig. 7 is “stretched” towards higher lifetimes, as could be anticipated

### Reliability Updating for the System Model of the Entire Bridge Structure by Enhanced Monte Carlo Simulation

The analysis so far has effectively been relevant for only a single “spot” or “component”. As most deterioration processes are spatially distributed in structures and the deterioration processes at different locations in a structure are correlated, such an analysis should be performed considering the structure as a whole.<sup>10-15</sup> Hence, a more realistic model corresponds to analysis of the whole bridge structure, which implies that assessment of the corresponding system reliability needs to be made.

This calls for more efficient simulation methods, and in the present study the enhanced Monte Carlo simulation

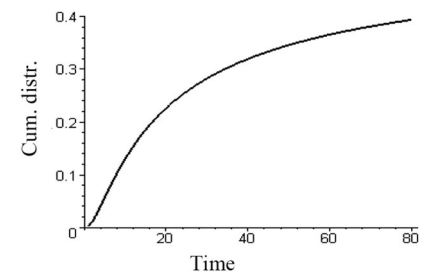


Fig. 6: Probability distribution of the lifetime of the superstructure corresponding to the input statistical models given in Table 3

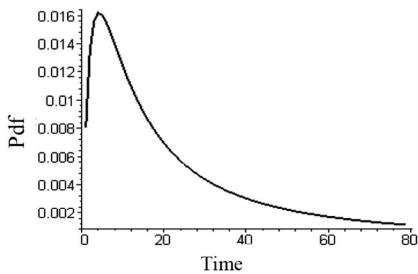


Fig. 7: Probability density function obtained by differentiation of the distribution function in Figure 5

technique is applied. In general, this approach is based on the introduction of a scaling parameter  $\lambda$  for the mechanical limit state function (which here corresponds to the critical level of chloride concentration at the steel reinforcement). A scaling factor of 1.0 corresponds to the “true” failure function, while a value of less than one leads to higher failure probabilities (i.e. a less reliable structural system). This corresponds to the physical situation that the critical level of the chloride concentration is scaled down such that corrosion will start at a lower level (accordingly leading to a higher failure probability). Similar scaling is also introduced for the “observation function”, which represents additional information that has become available based on for example monitoring or inspection of the structure. Within the present context, the observation function is associated with the measurement of a certain chloride concentration at the surface of the structure.

In the following, the main purpose is to assess the effect of correlation on the system reliability in the light of updating procedures. Some simplifications are introduced with respect to the relevant number of random parameters, but the main system effects are still believed to be reflected in a proper way.

#### Simplified System Model of the Bridge Superstructure

The bridge superstructure is considered to consist of 3 sites (i.e. components) in the transverse direction and 30 segments in the longitudinal direction, giving a total of  $3 \times 30 = 90$  components. In the transverse direction, each component represents the chloride ingress for one of the “faces” of the box girder, namely the windward face, the downward face and the

leeward face. The roadway itself is not included as the surface chloride concentration is much lower for this part than for the other surfaces. In the longitudinal direction, each component represents a certain length segment of each of the faces.

A simplified analysis is applied wherein only the surface concentration is represented as a random variable (while the other random variables are modelled as deterministic quantities equal to the mean values of the corresponding variables in the table above). The initial chloride concentration at the steel armour is set to zero.

For this purpose, the surface chloride concentration is represented by a mean value of 0.140% and a standard deviation of 0.028%. (Instead of a log-normal model, a Gaussian model is applied that is truncated at a value of 0.100% for the surface concentration.) These values correspond to a situation where the failure probability for both a single component and the entire bridge system is much smaller than in the previous case where models based on full-scale observations for the particular bridge were applied.

Presently, identical values are applied for the surface concentration of all “components” and accordingly the failure functions are the same for all components. However, the concentrations at different sites are assumed to be completely independent from each other, which implies that 90 independent random variables are to be sampled.

#### Reliability Updating Based on Inspection of the Surface Concentration

First, a system reliability analysis is performed based on the assumptions described above, with the system failure probability evaluated for a time in operation of  $t = 60$  years. The corresponding failure probability is shown as a function of the scaling parameter in Fig. 8 for the case where *no additional information from monitoring or inspection is available*. The 95% confidence band is also shown, as represented by the upper and lower curves.

The corresponding estimated failure probability for the system with 90 components (i.e. for the scaling parameter  $\lambda = 1.0$ ) is computed as  $5.72e-4$ , with

a 95% confidence interval of  $(4.78e-4, 6.58e-4)$ . This implies that the coefficient of variation for the estimated failure probability is around 5%. The total number of samples is 32 000, which corresponds to a reduction by a factor of around ten compared to the number required for the same level of accuracy in a crude Monte Carlo simulation.

It is next assumed that the surface concentrations for half the components are found to be lower than the mean value plus two standard deviations, that is 0.196%. The results based on an enhanced Monte Carlo simulation for the new updated failure probability at  $t = 60$  years are shown in Fig. 9 for increasing values of the scaling parameter (which is applied for both the failure function and the observation function).

The estimated failure probability for the system with 90 components (i.e. for the scaling parameter  $\lambda = 1.0$ ) is now found to be  $5.26e-4$ , with a 95% confidence interval of  $(3.53e-4, 6.93e-4)$ . This implies that the coefficient of variation for the estimated failure probability is around 15%. The total number of samples is now 16 000, which corresponds to a reduction by a factor of six compared to the number required (i.e. 100 000 samples) for a crude Monte Carlo simulation.

It is next assumed that the surface chloride concentration is observed to be less than the critical value of 0.18% for half the components (Fig. 10). The estimated failure probability for the system with 90 components (i.e. for the scaling parameter

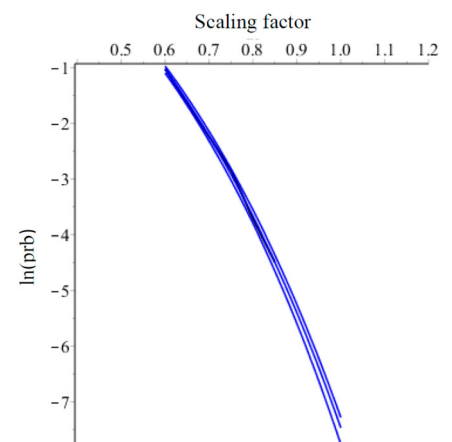


Fig. 8: Natural logarithm of the failure probability  $\ln(\text{prb})$  as a function of the scaling parameter at a time of  $t = 60$  years without any additional information from monitoring or inspection being available

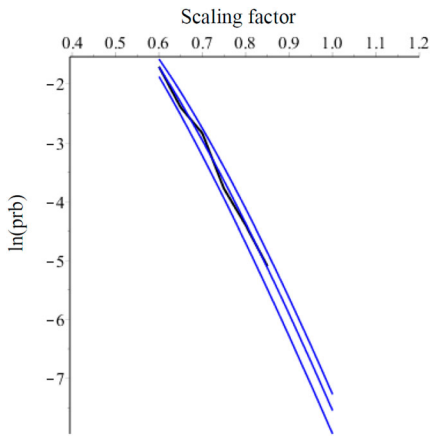


Fig. 9: Natural logarithm of failure probability  $\ln(\text{prb})$  as a function of the scaling parameter for the system with 90 components subjected to chloride ingress; failure probability at  $t=60$  years for the case that the surface concentrations for half the components are found to be smaller than 0.196%

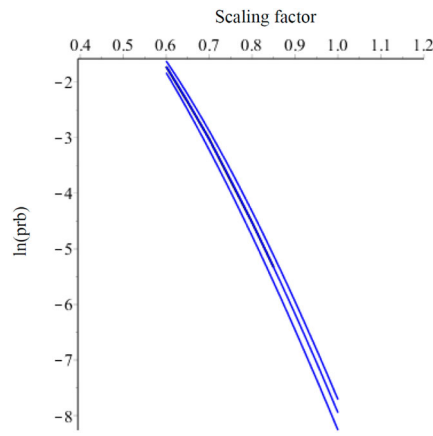


Fig. 10: Natural logarithm of failure probability  $\ln(\text{prb})$  as a function of the scaling parameter for the system with 90 components subjected to chloride ingress; failure probability at  $t=60$  years for the case that the surface concentrations for half the components are found to be smaller than 0.180%

$\lambda=1.0$ ) is now found to be  $3.52e-4$ , with a 95% confidence interval of  $(2.57e-4, 4.48e-4)$ . This implies that the coefficient of variation for the estimated failure probability is again around 15%. The total number of samples is 20 000, which still corresponds to a reduction by a factor of six compared to the number required for a crude Monte Carlo simulation.

For the present analysis, independence between the “components” is assumed. The corresponding effect of having additional information on the resulting system reliability is a reduction of the failure probability by roughly a factor of three. If correlation between the components was introduced, the effect would be much more pronounced. If full correlation was applied, this would essentially mean that there would only be a single component in the system, rather than 90. Accordingly, the failure probability would be reduced by a similar factor, even for the case in which only a single component was inspected.

## Conclusions

In the present paper, probabilistic models based on full-scale measurements from the Gimsøystraumen Bridge are addressed. These models apply to the diffusion coefficient, the chloride surface concentration and the concrete cover. Based on these models—and on supplementary models for other parameters affecting chloride diffusion—probabilistic

lifetime calculations are performed. The statistical analysis confirms the operators’ concern that the resistance of the bridge with respect to chloride ingress was not adequate, which is also reflected by repair and protective actions being initiated.

A system reliability analysis method was introduced and subsequent reliability updating was performed by means of enhanced Monte Carlo simulations. As a general observation, it was found that the computational effort (as measured by CPU time) was typically reduced by a factor of around six.

There are clearly multiple future research topics that need to be addressed, including: (a) the effect of the correlation between the system components in connection with the updated reliability for a range of additional information scenarios, (b) the effect of non-identical system components for example due to variations in means and standard deviations across the bridge dimensions, (c) the combination of parallel and series system models of bridge structures, and (d) ultimate limit state criteria in addition to serviceability criteria.

## Nomenclature

$c_i$	Initial chloride concentration at distance $x$ from concrete surface
$c_s$	Chloride concentration at surface (assumed to be constant with time)
$c(x,t)$	Chloride concentration at location $x$ at time $t$
$D$	Chloride diffusion coefficient
$E(.)$	Expected value

$k$	Cross-section incremental index
$\lambda$	Scaling parameter for limit state function
$N$	Total number of cross-sections
$P(.)$	Probability of event in parenthesis
$R_{i,i+k}$	Autocorrelation function corresponding to value of quantity $x$ at cross-section $i$ and $i+k$
$R^2$	Regression coefficient
$t$	Time
$x$	Distance from concrete surface

## Acknowledgements

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